Generating function for prime numbers

In this document we will describe how to build a generating function for prime numbers.

Prime numbers are not a well behaved sequence that could be generated by a relatively

simple generating function. So expect the level of complexity in prime numbers

to be reflected in this example.

If we denote by (x, y) the edges of a rectangle then x\*y is the surface of that rectangle.

lets assume that each integer is a surface that could be represented by one or more of

these surfaces with the condition x and y belong to the set of integers.

for example 7 is a prime number and it could be represented by (x, y) = (7, 1) or (1, 7)

4 is not a prime and it could be represented by (4, 1) (1, 4) (2, 2)

4 is a full square so it will have the extra (2, 2) form. Other numbers

such as 8 could be represented by (4, 2) (2, 4) (8, 1) and (1, 8)

In this way every prime number have exactly 2 forms. every full square have odd number

of forms, the other numbers have even number of forms above 2.

First observe that the generating function

> **series(1/(1-x),x,10);**

[Maple Math]

is streght forward expansion whit x powers 0,1,2....

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> **series(1/(1-x^2),x,10);**

[Maple Math]

The powers of the second series are 0,2,4,6... if we raise x to the power n = 7 we get.

> **series(1/(1-x^7),x,20);**

[Maple Math]

Now observe that the series of 1/(1-x^2) has 3 as the first missing element which is a prime number.

we add 1/(1-x^2) to 1/(1-x^3) the next gap wich is 5 is a prime number. but we can do better by

just adding all series and observe that all elements of the resulting series starting with 2 are primes.

> **series(sum(1/(1-x^i),i = 1..6000),x,60);**

[Maple Math]  
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[Maple Math]

The example above limits the search to 6000 integers and the primes within the set. All prime powers

start with 2. Example 2x^41 means 41 is a prime. 5x^16 start with an odd number because it is a full square.

All other numbers start with even numbers that represent the number of combinations of surfaces that could be

made from that number as explained above.

we can represent this frame by the function y = n/x where the graph of this function passes by the top right

corner of a rectangle x \* y. If n is a prime then the graph pass only by (x, y) = (1, n) and (n, 1).

This method of finding primes by generating functin is costly in terms of computing capacity. The function

y = n/x could be used to derive algorithms that looks for primes between two full square numbers of the

approximity of a given number.